

Full Name:
Student Number:

TOTAL POINTS: /14

Trent University MATH 4620 - Combinatorics
Instructor: Aras Erzurumluoğlu

Assignment 2 (due Oct 16th, until the end of the day)

READ ME: Attempt no more than 14 points worth of problems. When attempting the problems you are allowed to consult any resources for learning such as the lecture notes, textbooks, the internet, etc. In particular, you are encouraged to collaborate and brainstorm with your classmates. However, you are not allowed to copy solutions from the internet or from each other (or elsewhere). It is my assumption that everything you write as your solution is indeed your own work, meaning that at any time until the semester is over you will be able to produce the same work when asked (if need be) with minimal help.

Write your solutions in reasonable detail and use a precise mathematical language. You will lose points for imprecise solutions.

Problem 0) (0 points): Write a funny tercet (three lines of poetry) for each of the 5 rhyme schemes on 3 lines.

Problem 1) (2 points): A function f defined on positive integers is said to be **multiplicative** if $f(ab) = f(a)f(b)$ whenever a and b are coprime (i.e. $\gcd(a, b) = 1$). Prove that Euler's function ϕ is multiplicative.

Problem 2) (2 points): Find the number of red/blue circular sequences of length 8 that are not periodic. [You are not required (or even advised) to list them all, but you may do so if you wish.]

Problem 3) (2 points): Prove that $c(n, n-2) = 2\binom{n}{3} + 3\binom{n}{4}$.

Problem 4) (2 points): Recall that each permutation can be written as a product of disjoint cycles. How many permutations of $\{1, 2, \dots, 8\}$ are a disjoint product of one 1-cycle, two 2-cycles and one 3-cycle?

Problem 5) (1 point): Show that $c(n, k) \geq S(n, k)$ for all $n \geq 0, k \geq 0$.

Problem 6) (2 points): Let n be a positive squarefree integer with r distinct prime factors. In how many ways can you write n as a product of its divisors? (Count up to commutativity of multiplication – for example, if $n = 30$, don't count $30 = 2 \cdot 15$ and $30 = 15 \cdot 2$ separately.)

Problem 7) (3 points): Let $A = \{1, 2, 3, 4, 5, 6\}$. How many unordered pairs of subsets of A are there with the property that the two subsets have no elements in common? (For example, $\{\{1, 2, 5\}, \{3\}\}$ is such an unordered pair of subsets of A .)

Problem 8) (2 point): Prove that $S(n, 3) > 3^{n-2}$ for all $n \geq 6$.

Problem 9) (2 points): Find the number of integer solutions to $x_1 + x_2 + x_3 + x_4 \leq 17$ where $x_1 \geq 2$, $x_2 \geq 4$, $x_3 \geq 0$ and $x_4 \geq 1$. (Hint: Introduce x_5 .)